**<u>Using proper English and mathematical notation</u>**, state both parts of the Fundamental Theorem of Calculus, as well as the Net Change Theorem. **SCORE: _____ / 15 PTS**

SEE QUIZ 2 SOLUTIONS

Find $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{ne^{2+\frac{3i}{n}}}$ by finding the corresponding definite integral, and evaluating that integral. **SCORE: _____ / 15 PTS**

$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\Delta x)\Delta x$ WHERE $\Delta x = \frac{b-a}{n}$

$a+i\Delta x = 2+\frac{3i}{n} \longrightarrow a=2$ AND $\Delta x = \frac{3}{n} = \frac{b-a}{n} = \frac{b-2}{n}$

SO $b=5$

$f(a+i\Delta x)\Delta x = f\left(2+\frac{3i}{n}\right)\frac{3}{n} = \frac{3}{n}\frac{1}{e^{2+\frac{3i}{n}}}$

③ EACH

SO $f(x) = \frac{1}{e^x} = e^{-x}$

$\int_2^5 e^{-x}\,dx = -e^{-x}\Big|_2^5 = -(e^{-5}-e^{-2}) = e^{-2}-e^{-5}$

Prove that $\dfrac{\pi}{16} \le \displaystyle\int_{\frac{1}{2}}^{1} x \arcsin x \, dx \le \dfrac{3\pi}{16}$.

**SCORE: _____ / 15 PTS**

(3) $\frac{\pi}{6} \le \arcsin x \le \frac{\pi}{2}$ FOR $\frac{1}{2} \le x \le 1$

(2) EACH EXCEPT AS NOTED

$\frac{\pi}{6} x \le x \arcsin x \le \frac{\pi}{2} x$

$\int_{\frac{1}{2}}^{1} \frac{\pi}{6} x \, dx \le \int_{\frac{1}{2}}^{1} x \arcsin x \, dx \le \int_{\frac{1}{2}}^{1} \frac{\pi}{2} x \, dx$

$\int_{\frac{1}{2}}^{1} \frac{\pi}{6} x \, dx = \frac{\pi}{6} \left(\frac{1}{2} x^2\right)\Big|_{\frac{1}{2}}^{1} = \frac{\pi}{12}\left(1 - \frac{1}{4}\right) = \frac{\pi}{12} \cdot \frac{3}{4} = \frac{\pi}{16}$

$\int_{\frac{1}{2}}^{1} \frac{\pi}{2} x \, dx = \frac{\pi}{2} \left(\frac{1}{2} x^2\right)\Big|_{\frac{1}{2}}^{1} = \frac{\pi}{4}\left(1 - \frac{1}{4}\right) = \frac{\pi}{4} \cdot \frac{3}{4} = \frac{3\pi}{16}$

SO $\frac{\pi}{16} \le \int_{\frac{1}{2}}^{1} x \arcsin x \, dx \le \frac{3\pi}{16}$

The table gives the acceleration of a car (in feet per second$^2$) at various times (in seconds). **SCORE: _____ / 20 PTS**

At time $t = 3$ seconds, the velocity of the car was 21 feet per second.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a(t)$ | 4 | 2 | 1 | 3 | 2 | 5 | 4 | 2 | 0 | -1 | -3 | 0 | 3 | 5 |

[a] Write an expression involving an integral for the velocity of the car at $t = 11$ seconds.

(2) EACH EXCEPT AS NOTED

$v(11) - v(3) = \int_3^{11} v'(t)\,dt = \int_3^{11} a(t)\,dt$

so $v(11) = v(3) + \int_3^{11} a(t)\,dt = 21 + \int_3^{11} a(t)\,dt$

[b] Estimate the velocity of the car at $t = 11$ seconds using [a], 4 subintervals, and midpoints.

MIDPOINTS 4 6 8 10

3 5 7 9 11

$\Delta t = \frac{11-3}{4} = 2$

$21 + \int_3^{11} a(t)\,dt$

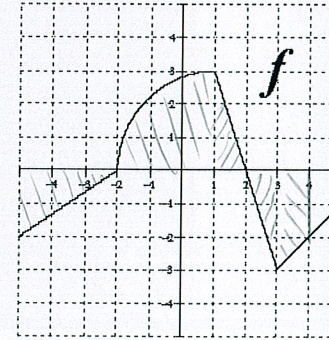
$\approx 21 + (a(4) + a(6) + a(8) + a(10))\Delta t$

(6)

$= 21 \frac{ft}{sec} + (2 + 4 + 0 + -3)\frac{ft}{sec^2}(2\ sec) = (21 + 6)\frac{ft}{sec} = 27 \frac{ft}{sec}$

Let $g(x) = \int_4^x f(t)\,dt$, where $f$ is the function whose graph is shown on the right. **SCORE: _____ / 25 PTS**

**NOTE: The graph of $f$ consists of a line, an arc of a circle, and two more lines.**




[a] Find $g(-5)$.

(2) EACH EXCEPT AS NOTED

$\int_4^{-5} f(t)dt = -\int_{-5}^{4} f(t)dt$

$= -\left[\frac{1}{4}\cdot\pi\cdot 3^2 + \frac{1}{2}\cdot 3\cdot 1 - \left(\frac{1}{2}\cdot 3\cdot 2 + \frac{1}{2}\cdot 1\cdot 3 + \frac{2+3}{2}\cdot 1\right)\right]$

$= -\left[\frac{9\pi}{4} + \frac{3}{2} - \left(3 + \frac{3}{2} + \frac{5}{2}\right)\right] = \frac{11}{2} - \frac{9\pi}{4}$ (3)

[b] Find $g'(1)$. **Explain your answer very briefly.**

$g'(1) = f(1) = 3$

[c] Find the $x-$coordinates of all inflection points of $g$. **Explain your answer very briefly.**

$g' = f$ CHANGES FROM INCREASING TO DECREASING @ $X=1$

DECREASING TO INCREASING @ $X=3$

$$\int z^5 \sqrt{1+z^2}\, dz$$

(3) $u = 1+z^2 \rightarrow z^2 = u-1$

$\frac{du}{dz} = 2z \rightarrow dz = \frac{du}{2z}$

$z^5\sqrt{1+z^2}\,dz = z^5\sqrt{1+z^2}\,\frac{du}{2z}$

$= \frac{1}{2} z^4 \sqrt{1+z^2}\,du$

(3) (3) $= \frac{1}{2}(u-1)^2\sqrt{u}\,du$ (2)

$= \frac{1}{2}(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}})du$

(3)

$\int \frac{1}{2}(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$

(3) $= \frac{1}{2}\left(\frac{2}{7}u^{\frac{7}{2}} - \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right) + C$

(3) $= \frac{1}{7}(1+z^2)^{\frac{7}{2}} - \frac{2}{5}(1+z^2)^{\frac{5}{2}}$

$+ \frac{1}{3}(1+z^2)^{\frac{3}{2}} + C$ (2)

$$\int_{-1}^{1} (y^2 - \csc^2 y)\, dy$$

$\csc^2 y$ IS NOT CONTINUOUS

(3) @ $y = 0 \in [-1, 1]$

SO FTC 2 DOES NOT APPLY

(2)

$$\int \frac{5x-10x^2}{\sin^2(4x^3-3x^2-1)}\,dx$$

(3) $u = 4x^3 - 3x^2 - 1$

$\frac{du}{dx} = 12x^2 - 6x \rightarrow dx = \frac{du}{12x^2-6x}$

$= \frac{du}{6(2x^2-x)}$

$\frac{5x-10x^2}{\sin^2(4x^3-3x^2-1)}\,dx$

$= \frac{-5(2x^2-x)}{\sin^2(4x^3-3x^2-1)} \frac{du}{6(2x^2-x)}$

(4) $= -\frac{5}{6} \csc^2 u\, du$ (3)

$\int -\frac{5}{6} \csc^2 u\, du$

$= \frac{5}{6} \cot u + C$ (3)

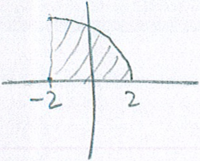
(3) $= \frac{5}{6} \cot(4x^3-3x^2-1) + C$ (2)

$$\int_{-2}^{2} \left(5\sqrt{16-(t+2)^2} - t\cos t^3\right) dt$$

$$= \int_{-2}^{2} 5\sqrt{16-(t+2)^2}\, dt - \int_{-2}^{2} t\cos t^3\, dt$$

(3)

~~CIRCLE~~
CENTER $(-2, 0)$
RADIUS 4



(3)

$$(-t)\cos(-t)^3$$
$$= -t\cos(-t^3)$$
$$= -t\cos t^3$$

(3)

$$= 5 \cdot \tfrac{1}{4}\pi 4^2 + 0$$
$$= 20\pi$$

(2)

INTEGRAND IS ODD + CONTINUOUS,

(2)

SO INTEGRAL $= 0$

(2)